

Ex 1.3 pages 19 & 20

(17) An apple farm

Solution:-

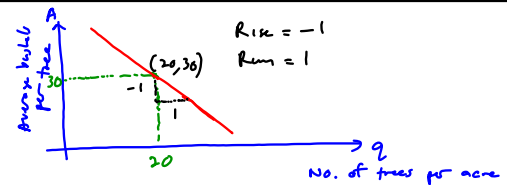
we are told that when $q=20$

then Average bushels per tree = 30

we are also told that

when no. of trees increase by 1

then average bushels per tree decrease by 1

When no. of trees increase by 1
Average bushels per tree decrease by 1

$$\text{So slope, } m = \frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$$

This straight line passes $(q, A) = (20, 30)$

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So Equation of the straight line is

$$A - A_1 = m(q - q_1)$$

$$\Rightarrow A - 30 = -1(q - 20)$$

$$\Rightarrow A - 30 = -q + 20$$

$$\Rightarrow A = -q + 20 + 30$$

$$\Rightarrow A = 50 - q$$

 $\therefore$ Average bushels per tree when there are q trees per acre is $50 - q$

So total bushels per acre,

$$Y = (\text{Average bushels per tree})(\text{No. of trees})$$

$$\Rightarrow Y = (50 - q)(q) \quad \checkmark$$

$$\therefore Y = 50q - q^2 \quad \checkmark$$

To find the domain, we need

$$Y \geq 0 \quad \text{and} \quad q \geq 0$$

$$\text{So we need } (50 - q)(q) \geq 0 \quad \text{and} \quad q \geq 0$$

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$$\Rightarrow 50 - q \geq 0 \quad \text{and} \quad q \geq 0$$

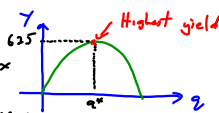
$$\Rightarrow q \leq 50 \quad \text{and} \quad q \geq 0$$

$$\Rightarrow 0 \leq q \leq 50 \quad \text{This is the domain.}$$

$$\therefore Y = (50 - q)(q) ; 0 \leq q \leq 50$$

$$\Rightarrow Y = -q^2 + 50q$$

Highest yield occurs at the vertex



$$q^* = h = -\frac{b}{2a} = -\frac{50}{2(-1)} = 25 \text{ trees per acre}$$

$$\text{Total max. yield is } Y^* = (50 - 25)(25) = 625 \text{ bushels per acre.}$$

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1.4 General Polynomial Functions

A polynomial takes the form

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

where n is a non-negative integer (whole number)and the highest power of x with a non-zero coefficient is the degree of the polynomial

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Examples

- ① $y = 2x - 3$ is a polynomial of degree 1
- ② $y = 3x^2 - 2x + \frac{1}{2}$ is a polynomial of degree 2
- ③ $y = 0.1x^3 - 0.2x^2 + 3$ is a polynomial of degree 3
- ④ $y = x^4 + x^{2.5} + 2x^2$ is NOT a polynomial
- ⑤ $y = x^2 + x^{-2}$ is NOT a polynomial
- ⑥ $y = 2$ is a polynomial of degree 0
 $= 2x^0 \leftarrow$ assume it is like this

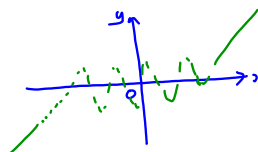
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Graphs of a polynomial

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

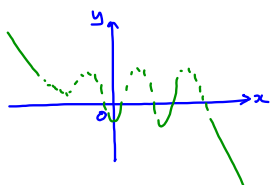
Four cases :-

- ① When n is odd and a_n is +ve



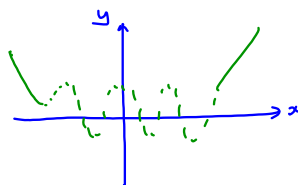
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- ② When n is odd and a_n is -ve



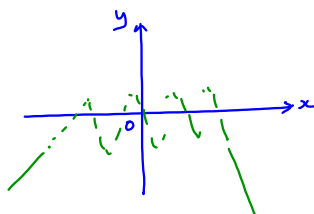
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- ③ When n is even and a_n is +ve



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- ④ When n is even and a_n is -ve



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- ② If the total cost function is

$$C = -0.0012q^3 - 0.05q^2 + 0.022q + 8500$$

determine the fixed and variable costs.

Solution:-

we need to write C as

$$C = F + vq$$

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$$\text{Given } C = -0.0012q^3 - 0.05q^2 + 0.022q + 8500$$

$$\Rightarrow C = 8500 - 0.0012q^3 - 0.05q^2 + 0.022q$$

$$\Rightarrow C = \underbrace{8500}_F + \underbrace{(-0.0012q^3 - 0.05q^2 + 0.022q)}_v$$

Comparing with

$$C = F + vq$$

we have

Fixed cost, $F = \$8,500$ and Variable cost, $v = -0.0012q^3 - 0.05q^2 + 0.022$

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1.5 Rational Functions

A rational function is the ratio of two polynomials, i.e.

$$y = \frac{g(x)}{h(x)}$$

when $g(x)$ and $h(x)$ are both polynomials
and $h(x) \neq 0$

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Many types of rational functions

① Linear-to-Linear

$$y = \frac{ax+b}{x+c} \quad \leftarrow \text{Linear}$$

How do we call it?
Numerator-to-Denominator

what happens if we get

$$y = \frac{3x+2}{4x-1}$$

$$\Rightarrow y = \frac{3x+2}{4(x-\frac{1}{4})} = \frac{1}{4} \left(\frac{3x+2}{x-\frac{1}{4}} \right) = \frac{\frac{1}{4}(3x+2)}{x-\frac{1}{4}} = \frac{\frac{3}{4}x + \frac{2}{4}}{x-\frac{1}{4}}$$

$$= \frac{\frac{3}{4}x + \frac{1}{2}}{x-\frac{1}{4}}$$

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② Quadratic-to-Linear

$$y = \frac{ax^2+bx+c}{x+d} \quad \leftarrow \text{Quadratic}$$

③ Linear-to-Quadratic

$$y = \frac{ax+b}{x^2+cx+d}$$

④ Quadratic-to-Quadratic

$$y = \frac{ax^2+bx+c}{x^2+dx+e} \quad \dots \text{And other types.}$$

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All rational functions

$$y = \frac{g(x)}{h(x)} \quad \text{provided } \frac{g(x)}{h(x)} \text{ is irreducible}$$

can be rewritten as

$$y = s(x) + \frac{r(x)}{h(x)} \quad \text{where } \frac{r(x)}{h(x)} \text{ is proper.}$$

and $s(x)$ is a polynomialNote A proper rational is where

$$\deg(\text{numerator}) < \deg(\text{denominator})$$

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All rational functions

$$y = \frac{g(x)}{h(x)} \quad \text{provided } \frac{g(x)}{h(x)} \text{ is irreducible}$$

$$\Rightarrow y = s(x) + \frac{r(x)}{h(x)}$$

and we obtain the non-vertical asymptote

$$\text{as } y = s(x)$$

and the vertical asymptotes (if they exist)

are the solutions to

$$h(x) = 0$$

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General example :-
 Let us consider a quadratic-to-linear rational
 $y = \frac{ax^2+bx+c}{x+d}$ in irreducible form

Irreducible means
 nothing in the numerator
 and denominator cancel
 each other.

Domain is obtained by considering
 the original form.

Review :-
 $y = \frac{x^2-3x+2}{x-1}$
 is reducible
 $y = \frac{x^2-3x+2}{x-1} = \frac{(x-1)(x-2)}{x-1}$
 $= x-2$

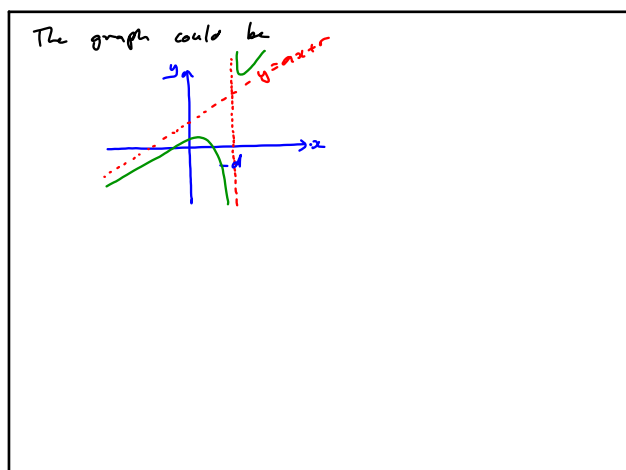
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$$y = \frac{ax^2+bx+c}{x+d}$$

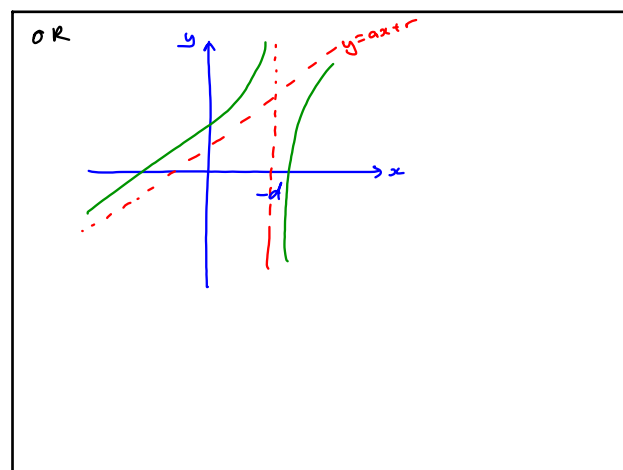
$$\Rightarrow y = ax+r + \frac{s}{x+d}$$

The non-vertical asymptote is
 $y = ax+r$
 and the vertical asymptote is the solution
 to $x+d=0$
 $\Rightarrow x=-d$ is the vertical asymptote

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② $y = \frac{2x+1}{x-4}$

we can perform a long division

$\therefore y = \frac{2x+1}{x-4} \Rightarrow y = 2 + \frac{9}{x-4}$

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OR If the denominator is linear, we
 could perform a synthetic division

$$y = \frac{2x+1}{x-4} \rightarrow x-4=0 \Rightarrow x=4$$

$\therefore y = \frac{2x+1}{x-4} \Rightarrow y = 2 + \frac{9}{x-4}$

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So we rewrite

$$y = \frac{2x+1}{x-4}$$

$$\Rightarrow y = 2 + \frac{9}{x-4}$$

$\therefore$ The non-vertical asymptote is

$$y = 2 \quad \leftarrow \text{Happens to be a horizontal asymptote}$$

and the vertical asymptote is given by

$$x-4=0 \Rightarrow x=4$$

To find the domain, we need

$$x-4 \neq 0$$

$$\Rightarrow x \neq 4$$

$$\Rightarrow \text{Domain is } x \in (-\infty, 4) \cup (4, \infty)$$

To sketch the graph, we need to know the axes intercepts.

① To find the x-intercepts, we assign $y=0$ and solve for x .

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$$\text{so } \frac{2x+1}{x-4} = 0$$

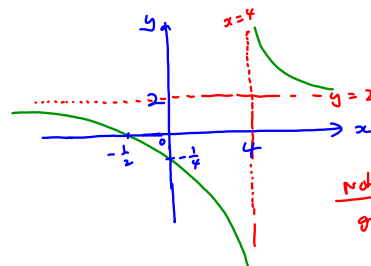
$$\Rightarrow 2x+1=0$$

$$\Rightarrow x = -\frac{1}{2} \quad \leftarrow \text{This is the x-intercept.}$$

② To find the y-intercept, we assign $x=0$ and calculate y

$$\text{so } y = \frac{2(0)+1}{0-4} = -\frac{1}{4} \quad \leftarrow \text{This is the y-intercept.}$$

Now, we draw the graph



Note:- This type of graph is called a hyperbola

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⑤ Determine the linear-to-linear rational function that has a horizontal asymptote at $y=5$, a vertical asymptote at $x=-3$, and goes through the point $(3, 8)$.

Solution:-

General function consisting of a polynomial and a proper rational is

$$y = s(x) + \frac{t(x)}{h(x)}$$

We are told that there is a non-vertical asymptote $y=5$

$$\Rightarrow y = s(x) \text{ is the non-vertical asymptote}$$

$$\text{so } s(x) = 5$$

We are also told that there is a vertical asymptote at $x=-3$. This is the solution to $h(x)=0$.

$$\text{so from } x = -3$$

$$\Rightarrow x+3=0$$

$$\Rightarrow h(x) = x+3 \quad \text{since } h(x)=0$$

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Up to now we have

$$y = s(x) + \frac{t(x)}{u(x)}$$

$$\Rightarrow y = 5 + \frac{t(x)}{x+3}$$

Since $\frac{t(x)}{x+3}$ is a proper rational, therefore

$$\deg(t(x)) < \deg(x+3)$$

$$\Rightarrow \deg(t(x)) < 1$$

$$\Rightarrow \deg(t(x)) = 0 \quad \therefore t(x) = \text{constant}$$

$$\Rightarrow y = 5 + \frac{c}{x+3}$$

To find c , we are told that the function passes $(3, 8)$ i.e. when $x=3$, then $y=8$

$$\text{so } 8 = 5 + \frac{c}{3+3}$$

$$\Rightarrow \frac{c}{6} = 8 - 5 = 3$$

$$\Rightarrow c = (6)(3) = 18$$

$$\Rightarrow y = 5 + \frac{18}{x+3} = \frac{5(x+3)}{x+3} + \frac{18}{x+3}$$

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$$\Rightarrow y = \frac{5(x+3) + 18}{x+3}$$

$$= \frac{5x + 15 + 18}{x+3}$$

$$\Rightarrow y = \frac{5x + 33}{x+3}$$

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8 (iii) Find equilibrium in the Market
(clearing Price on demand $q \in [0, \infty)$)

$$\text{Supply: } p = \frac{40q + 120}{q + 20}$$

$$\text{Demand: } p = \frac{5q + 300}{q + 10}$$

Solution:-

Equilibrium occurs when supply = demand

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$$\Rightarrow \frac{40q + 120}{q + 20} = \frac{5q + 300}{q + 10}$$

$$\Rightarrow (40q + 120)(q + 10) = (5q + 300)(q + 20)$$

$$\Rightarrow 40(q + 3)(q + 10) = 5(q + 60)(q + 20)$$

$$\Rightarrow \frac{40}{5}(q^2 + 13q + 30) = q^2 + 80q + 1200$$

$$\Rightarrow 8(q^2 + 13q + 30) = q^2 + 80q + 1200$$

$$\Rightarrow 8q^2 + 104q + 240 = q^2 + 80q + 1200$$

$$\Rightarrow 8q^2 + 104q + 240 - q^2 - 80q - 1200 = 0$$

$$\Rightarrow 7q^2 + 24q - 960 = 0$$

$$\Rightarrow q = \frac{-24 \pm \sqrt{24^2 - 4(7)(-960)}}{2(7)}$$

$$= \frac{-24 \pm \sqrt{27456}}{14}$$

$$\Rightarrow q \approx \frac{-24 \pm 165.6985}{14}$$

$$\Rightarrow q \approx \frac{-24 - 165.6985}{14} \text{ or } q \approx \frac{-24 + 165.6985}{14}$$

Quadratic Formula:-
 $q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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Since $q \geq 0$, we reject

$$q = \frac{-24 - 165.6985}{14}$$

and accept only

$$q = \frac{-24 + 165.6985}{14}$$

$$\Rightarrow q \approx 10.1213$$

$$\Rightarrow q \approx 10$$

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To find the market clearing price Supply function

$$MCP = \frac{40(10.1213) + 120}{10.1213 + 20} \approx \$17.42$$

OR

$$MCP = \frac{5(10.1213) + 300}{10.1213 + 10} \approx \$17.42$$
Demand function

Note:- Use the value of q before rounding to find the MCP.

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